Calculus HW 4-7-2017 5.2 1, 3, 5, 9, 11, 15, 17, 23

1. $f(x) = x^2+2x-1$ on $[0,1]$

$m = \dfrac{f(1)-f(0)}{1-0} = (1)^2+2(1)-1-((0)^2+2(0)-1)$

$m = 1+2-1-0-0+1 = 3$

$f'(x) = 2x+2 \qquad m = f'(x) \qquad 3 = 2x+2 \quad x = \frac{1}{2}$

(a) the function satisfies the hypotheses of the MVT.

(b) $c = \frac{1}{2}$

3. $f(x) = x^{1/3}$ on $[-1,1]$

$f(x)$ is continuous on the closed interval $[-1,1]$

$f'(x) = \frac{1}{3}x^{-2/3} = \dfrac{1}{3\sqrt{x^2}} \qquad f'(0) = \text{undefined}.$

$f(x)$ is not differentiable on the open interval $(-1,1)$. The function does not satisfy hypothesis 2 of the MVT

5. $f(x) = \sin^{-1}x$ on $[-1,1]$ $\qquad f(1) = \sin^{-1}(1) = \frac{\pi}{2}$

$f(x)$ is continuous on $[-1,1]$ $\qquad f(-1) = \sin^{-1}(-1) = -\frac{\pi}{2}$

$f'(x) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad f(x)$ is differentiable on $(-1,1)$

$m = \dfrac{f(1)-f(-1)}{1--1} = \dfrac{\frac{\pi}{2}-\frac{-\pi}{2}}{2} = \dfrac{\pi}{2} \qquad m = f'(x)$

$\frac{\pi}{2} = \dfrac{1}{\sqrt{1-x^2}}$ using calculator $x \approx -0.771$
$x \approx 0.771$

$c \approx -0.771$ or $c \approx 0.771$

5.2 9. $f(x) = x + \frac{1}{x}$ on $\frac{1}{2} \le x \le 2$

$f(2) = 2 + \frac{1}{2} = 2.5 \quad f\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\left(\frac{1}{2}\right)} = \frac{1}{2} + 2 = 2.5$

(a) secant line $\overline{AB}$ equation: $y = 2.5$

(b) $f'(x) = 1 + \frac{-1}{x^2} = \frac{x^2 - 1}{x^2} \quad f'(1) = 0$

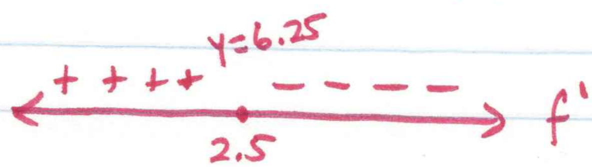
$f(1) = 1 + \frac{1}{1} = 2$ tang. line equation: $y = 2$

11. $(t, d) = $ (hours, distance) $\quad (t, d) = (2, 159)$

$(t, d) = (0, 0)$ secant slope is $m = \frac{159}{2} = 79.5$ mph

The MVT says that it is quarenteed that there was at least one instance during the two-hour time interval that the trucker was traveling at a speed of 79.5 mph.

15. $f(x) = 5x - x^2 \quad f'(x) = 5 - 2x \quad 0 = 5 - 2x \quad x = 2.5$



$f(2.5) = 12.5 - 6.25 = 6.25$

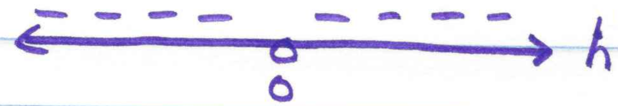
(a) $f(x)$ has an abs. max of $y = 6.25$ at $x = 2.5$ since $f'(x)$ changes from Pos. to Neg.

(b) $f(x)$ is inc. on $(-\infty, 2.5)$ since $f'(x) > 0$

(c) $f(x)$ is dec. on $(2.5, \infty)$ since $f'(x) < 0$

5.2 17. $h(x) = \frac{2}{x}$ Domain: $(-\infty, 0) \cup (0, \infty)$

$h'(x) = -\frac{2}{x^2}$



(a) $h(x)$ has no extrema
(b) $h(x)$ is never increasing
(c) $h(x)$ is decreasing on $(-\infty, 0) \cup (0, \infty)$ since $h'(x) < 0$

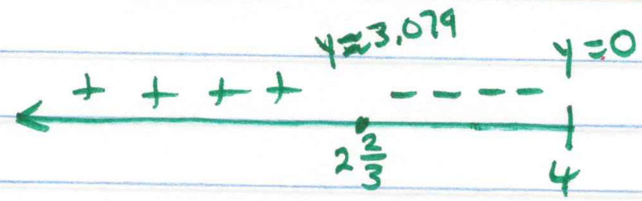
23. $f(x) = x\sqrt{4-x}$ Domain: $(-\infty, 4]$

$f(4) = 0$ $f'(x) = \sqrt{4-x} + x \cdot \frac{1}{2}(4-x)^{-\frac{1}{2}}(-1)$

$f'(x) = \sqrt{4-x} - \frac{x}{2\sqrt{4-x}} = \frac{2(4-x) - x}{2\sqrt{4-x}}$

$f'(x) = \frac{8-3x}{2\sqrt{4-x}}$ $0 = \frac{8-3x}{2\sqrt{4-x}}$ $0 = 8-3x$

$x = \frac{8}{3}$ $f'(2\frac{2}{3}) = 0$



$f(2\frac{2}{3}) \approx 3.079$

(a) $f(x)$ has an abs. max. of $y \approx 3.079$ at $x = 2\frac{2}{3}$ since $f'(x)$ changes from Pos. to Neg.
$f(x)$ has a local min of $y = 0$ at $x = 4$ since $x = 4$ is an endpoint and $f'(x) < 0$ on $(\frac{8}{3}, 4)$
(b) $f(x)$ is increasing on $(-\infty, \frac{8}{3})$ since $f'(x) > 0$
(c) $f(x)$ is decreasing on $(\frac{8}{3}, 4)$ since $f'(x) < 0$